



PROBABLY PROBABILITY

Understanding the idea of likelihood

6th Grade Lesson Plan: Probably Probability

Overview

This series of lessons was designed to meet the needs of gifted children for extension beyond the standard curriculum with the greatest ease of use for the educator. The lessons may be given to the students for individual self-guided work, or they may be taught in a classroom or a home-school setting. This particular lesson plan is primarily effective in a classroom setting. Assessment strategies and rubrics are included. The lessons were developed by Lisa Van Gemert, M.Ed.T., the Mensa Foundation's Gifted Children Specialist.

Introduction

Probability is practical math that is interesting and useful at the same time. It is easy for youth to engage in fairly complex exercises in probability because it speaks to them. This lesson explores some fundamentals of probability and its application in the "real" world. For an interesting take on why this type of mathematics is so important, watch Arthur Benjamin's short talk What Math Students Should be Learning, available through the TED site at <http://ow.ly/ohIUN>.

Learning Objectives

After completing the lessons in this unit, students will be able to:

- Understand the idea of likelihood.
- Manipulate the mathematical formulas for probability and relative frequency.
- Calculate possible outcomes.
- Make predictions of probability of an event.

Preparation

- Gather materials:
 - ▶ Penny
 - ▶ Dice
 - ▶ Pen or pencil
- Read through lesson plan and print it, preferably in color.
- Keep the Assessment Key for your own use.

Lesson 1: Introduction

Whether you know it or not, you are a fortune teller of sorts. Every day, all day, you are constantly predicting what will happen:

- You choose clothes based on what you think the weather will be (or, be honest, what you have that's clean regardless of the weather).
- You choose what table to sit at in the cafeteria based on where you think your friends will sit.
- You choose and choose and choose, and every choice is a prediction of how likely you think an event or series of events is to happen.

We can actually measure how likely it is for something to happen, and that measurement is called probability.



Lesson 2: Probability and relative frequency

Let's start with dice. Take a die (make sure it's fair, not weighted or "funny" in any way). What are the odds (the probability) of rolling a 3 if you roll the die one time? Hopefully you figured out that it is one in six because there are six sides to the die, and one of those sides has three dots.

Let's try it. Take the die and roll it 100 times, recording your results below. Calculate the percentage of each result (for example, if you rolled a 2 17 times, that would be $17/100$, or 17 percent).

Dots on die	Times rolled (use tally marks)	Percentage of rolls
1		
2		
3		
4		
5		
6		



We would expect that the percentages for each number would hover around 16 or 17, which is $1/6$ or .1666. This is probability in a nutshell.

Now guess what the percentage would be if you added up the percentages of the rolls of only the odd-numbered sides. When you add up those rolls, does the percentage come close to your guess?

Probability is the ratio of the times an event is likely to occur divided by the total possible events.

In the case of our die, there are six possible events, and there is one likely event for each number with each roll, or $1/6$.

If there were no dots on any of the sides, the probability of rolling a 3 would be zero because there would be no 3 and no other dots either, giving us this ratio: $0/6$. If every side had three dots, the probability of rolling a 3 would be 1 because it would be $6/6$, or 1. So, probability is expressed as a number somewhere between 0 (not gonna happen) and 1 (definitely going to happen), with ratios closer to 1 being most likely.

Let's put it in formula form:

Using the formula:

- What is the probability of getting heads on a coin toss?

- If you go to school

Monday through Friday and you know the cafeteria is going to serve pizza two days that week, what is the probability that pizza will be served on any given day? It would be $\frac{2}{5}$, because there are two desired outcomes (pizza!) and five possible outcomes (days of the school week).

So far, we've just looked at things that could occur. What about looking at things that have actually happened? We call that relative frequency, and it has its own formula:

relative frequency:

$$\frac{T \text{ (times it happened)}}{R \text{ (times it was tried)}}$$

Think back to your die experiment above. The first formula gives you your expectation ($\frac{1}{6}$). The formula for relative frequency gives you the actual outcome. What was the relative frequency of rolling 5s in your die-rolling experiment?



Lesson 3: The law of large numbers

The more times we roll the die, the closer we will get to the outcome we expected ($1/6$). We call that the Law of Large Numbers — even if you don't get it to come out like you expect with a few tries, the more you do it, the closer you will come to the expectation.

Flip a penny 10 times and record how many times it lands on heads and how many times it lands on tails.

Heads: _____

Tails: _____

Now flip it 100 times and record your results.

Heads: _____

Tails: _____



Did you get closer to a $\frac{1}{2}$ ratio the second time? That's the Law of Large Numbers at work. Remember, the Law of Large Numbers tells us that the more times you repeat an experiment, the closer the relative frequency will come to the probability.

Lesson 4: Calculating possible outcomes

We have one more thing to learn before we move on. Let's figure out how to calculate the possible outcomes of an event. We've figured out the probability of simple events occurring, but what happens when the possible outcomes are harder to figure out?

When you are rolling one die or flipping one coin, it's simple to figure out possible outcomes, but it gets more complicated when you add in more dice or more coins.

Imagine that your parents pay you an allowance of 50¢ a week (they're so stingy!). Let's say you love nickels. What is the probability that your 50¢ will contain a nickel? We need to figure out all the possible ways to give someone 50¢.

Let's say that your parents don't ever use pennies or 50-cent pieces. Besides those, there are three possible coins to use: nickels, dimes and quarters. If your parents pay you in quarters, it's simple, right? Two quarters make 50¢. But what other possible combinations make 50¢? And how likely are you to get that nickel you want?

Let's set it up in the table below. First, across the top we'll list the possible coins. Next, we'll start listing possible combinations by listing the greatest possible number of that type of coin and then decreasing that by one on the next line. Once we've gotten to zero of that coin, let's move to the highest level possible of the next highest coin. Sound confusing? It's not once you get going. Let's try it:



	Quarters (25¢)	Dimes (10¢)	Nickels (5¢)
1	2	0	0
2	1	2	1
3	1	1	3
4	1	0	5
5	0	5	0
6	0	4	2
7	0	3	4
8	0	2	6
9	0	1	8
10	0	0	10

First, quarters. We list two quarters, and that makes 50¢ by itself, so the columns for dimes and nickels are zero. There are no other possible combinations with two quarters, so the next line lists one quarter. Remember, we are going from largest to smallest, so first we'll try one quarter and the greatest number of dimes possible, which is two. There are two other possibilities with one quarter, so we'll list those on the next lines and then the next lower number of dimes, which is one. That means we'll need three nickels because we've always got to add up to fifty cents.

Now we're out of possibilities that use quarters, so we move on to dimes. The most dimes we could have is five, so we start with that. We decrease that number by one on each line. For every dime we take away, we have to add two nickels, so notice that the nickels increase by two each line. We end up with 10 possible combinations of coins.

So, how many of the 10 possibilities contain nickels? Did you count eight? You're right! So let's put this in our probability formula:

$\frac{8}{10}$ = desired (or likely) outcomes

$\frac{10}{10}$ = possible outcomes

Does that seem like good odds to you? Not bad! What are the odds if you want a quarter? How many possibilities contained a quarter? Did you find four? So the odds of getting a quarter are:

$\frac{4}{10}$ = likely outcomes

$\frac{10}{10}$ = possible outcomes

Are you more likely to get a quarter or a nickel? All other things being equal (meaning your parents have a wide variety of coins and aren't out of dimes or something), you are far more likely to get a nickel than a quarter.



Assessment: Probability at work

Now's your chance to do some more experiments with probability using what you have learned.

1. Suppose you go to the store and there are six kinds of cereal. Your mom tells you to pick one. When she comes back, you ask her to guess which one you chose. Assuming she had no idea beforehand what your choice would be, how likely is it that she will guess the correct one?

2. Let's say you are playing a coin-tossing game with a friend. You toss the same coin 500 times, and 400 times it comes up heads. Would this be normal? What could explain it?



3. You are bored one day, so you start rolling a die. You roll it 10 times and you get a 6 eight of the times. You think this is strange, so you keep rolling. You roll 100 more times and only get eight more 6s, leaving a total of 16. What is the rule that accounts for this scenario?

4. What is the relative frequency of the final total of the 6's rolled in Question 3?

5. Someone hands you a standard deck of 52 cards. There are four queens. What is the likelihood that you will draw a queen from the deck the first try?

6. Let's say you earn \$20 doing chores (lots of them), and you're being paid in cash. Create a table of possible outcomes that shows the possible combinations of \$10's, \$5's and \$1's that would total your \$20. (Look back at the table we did before to see how to set this problem up.)

7. You and a friend (who has no understanding of probability) are at the mall on a hot Tuesday afternoon in the middle of the summer. She offers to buy you a snow cone if you can guess the types of dollar bills she has in her wallet in fewer than three guesses. She tells you she has \$20 and none of the bills are \$1's or a \$20. If you don't guess correctly by the third try, you have to give her a quarter. Are your odds of guessing correctly greater than $\frac{1}{2}$?

Assessment key

1. $1/6$
2. No, it would not be normal. Most likely, either a trick is being played or there is something wrong with the coin.
3. The Law of Large Numbers
4. $16/100$
5. $4/52$
6. Answers will vary. If the student sets the table up like the example in the lesson, it will look something like this:

	\$10 Bills	\$5 Bills	\$1 Bills
1	2	0	0
2	1	2	0
3	1	1	5
4	1	0	10
5	0	4	0
6	0	3	5
7	0	2	10
8	0	1	15
9	0	0	20

7. Since she has no \$1s or a \$20, only three possible combinations equal \$20, and she's giving you three guesses. Therefore, you have a greater than $\frac{1}{2}$ chance of guessing correctly.

	\$10 Bills	\$5 Bills
1	2	0
2	1	2
3	0	4